[09-01-21-NEM11]

Exponents and Logarithms

[1.1]

1. Without using a calculator, evaluate:

(a) $(2^3 \times 3^2)^0$ **(b)** $8^{\frac{1}{2}} \times 8^{\frac{1}{6}}$ **(c)** $(2\sqrt{3})^4$

(d) $\dfrac{9^{\frac{1}{3}} \times 3^{\frac{1}{3}}}{6}$ **(e)** $25^{\frac{1}{4}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{6}}$ **(f)** $\dfrac{3^{\frac{1}{3}} \times 3^0 \times 9^{\frac{1}{3}}}{27^{\frac{2}{3}}}$

2. Given $y = 2x^{\frac{3}{2}}$, find

(a) y when $x = 9$, **(b)** x when $y = 16$.

3. If $2^x = y$, express the following in terms of y.

(a) 2^{x+3} **(b)** 2^{2x-1} **(c)** $3(2^{2-x})$

(d) $4(2^3)^{1-x}$ **(e)** $8(4^{x-2})$ **(f)** $8^x - 4^{-x}$

4. If $3^x = y$ and $2^x = z$, express the following in terms of y and/or z.

(a) $2(9^{x+1})$ **(b)** 6^{x-1} **(c)** $9^{-x} \times 12^x$

5. Show that $(8x^2)^{8-r}\left(\dfrac{1}{2x}\right)^r = 2^{24-4r}(x^{16-3r})$.

[1.2]

1. Solve the following equations.

(a) $3^{2x} = 27$ (b) $4^x = 32$

(c) $(\sqrt{2})^{3x} = \frac{1}{8}$ (d) $\left(\frac{1}{9}\right)^{x+2} = 3$

(e) $4^x(5^{2x}) = 10$ (f) $5^x - 25 = 0$

(g) $7^{x^2-4} - 1 = 0$ (h) $8^x = 4^{x+1}$

(i) $9^x = (\sqrt{3})^{x+2}$ (j) $\frac{4^x}{2^{x-1}} = 8^{2-x}$

(k) $2^{x^2} = 4^{2(x-1)}$ (l) $7^{x^2} = 49^{6-2x}$

(m) $4^{x^2-6} - 16^{x+1} = 0$ (n) $3^{x^2} = \frac{9^{x+1}}{27^x}$

2. Given that $y = ax^n - 23$, and that $y = 4$ when $x = 3$ and $y = 220$ when $x = 9$, find the value of a and of n.

3. Solve the following simultaneous equations.

(a) $5^x(25^{2y}) = 1$ and $3^{5x}(9^y) = \frac{1}{9}$

(b) $2(4^x) = 32^y$ and $\frac{125^x}{25^y} = 625$

(c) $\frac{3^x}{9^y} = 27$ and $4^{2x}(2^{6y}) = \frac{1}{4}$

4. Show that $\sqrt[n]{2 \times 4^m} = 2^{\frac{2m+1}{n}}$. Hence find the value of m and of n which satisfy the equations $\sqrt[n]{2 \times 4^m} = 8$ and $\frac{27^m}{9^{n+1}} = 81$ simultaneously.

5. By using the substitution $y = 2^x$, find the value of x such that
$$3(2^{x-1}) = 2^x + 4.$$

6. By using the substitution $y = 3^x$, find the values of x such that

(a) $3^x = 4 - 3(3^{-x})$, (b) $3^{2x} - 3^{x+2} = 3^x - 9$.

7. By using appropriate substitutions or otherwise, solve the following equations.

(a) $5^{2x} - 6(5^x) + 5 = 0$ (b) $2^{2x} - 10(2^x) + 16 = 0$

(c) $2(16^x) - 5(4^x) + 2 = 0$ (d) $7^{x+1} - 2 = 2(7^x) + 3$

(e) $9^{x+1} + 1 = 10(3^x)$ (f) $6(3^{x-1}) = 3^4 - 3^x$

8. Show that the equation $2^{2x+1} = 3(2^x) + 2$ is satisfied by only one value of x.

9. Solve the following equations.

(a) $8\left(\frac{1}{2}\right)^x + 2 = 2^x$ (b) $4^x = 2^{\frac{8}{x}}$

(c) $3^{x+1}(9^{2-x}) = \frac{1}{3}$ (d) $4^{x+\frac{1}{2}} - 2^{x+3} = 8 + 7(2^x)$

(e) $\frac{5^{x+1} - 20}{10 + 5^x} = 3$ (f) $2^x(5^x) = \frac{1}{100}(10^{x-1})^4$

10. If $\frac{r^2}{4}(3x)^r\left(\frac{2}{9x^2}\right)^{6-r}$ can be simplified to $\frac{k}{x^3}$, find the values of the constants r and k.

*11. Solve the simultaneous equations $64(4^y) = 16^x$ and $3^y = 4(3^{x-2}) - 1$.

*12. By using an appropriate substitution, find the value of x for which
$$x^{\frac{3}{2}} - 8x^{-\frac{3}{2}} = 7.$$

[1.3]

1. Convert the following to logarithmic form.

(a) $2^4 = 16$ **(b)** $3^{-2} = \frac{1}{9}$

(c) $100 = 10^2$ **(d)** $a^3 = y$

(e) $2^x = p$ **(f)** $x^4 = 2 - k$

2. Convert the following to index form.

(a) $3 = \log_5 125$ **(b)** $-2 = \log_2\left(\frac{1}{4}\right)$

(c) $\log_4 64 = 3$ **(d)** $\log_x 3 = 4$

(e) $\log_3 y = n$ **(f)** $p + 1 = \log_2 (4y)$

3. Check whether the logarithm $\log_x (5 - 2x)$ is defined when:

(a) $x = 2$ **(b)** $x = 0.5$ **(c)** $x = 3$

(d) $x = 2.5$ **(e)** $x = 1$ **(f)** $x = \sqrt{2}$

4. Solve the following equations.

(a) $\log_2 x = 3$ **(b)** $\log_x 9 = 2$

(c) $x = \log_4 8$ **(d)** $\log_3 (x - 2) = 1$

(e) $\log_2 (2x + 1) = -3$ **(f)** $\log_9 \sqrt{27} = x$

(g) $\log_x (6x - 8) = 2$ **(h)** $\log_x 8 = \frac{3}{2}$

5. Evaluate.

(a) $\log_4 4 - 3 \log_2 2$ **(b)** $\log_2 1 + 2 \log_5 5$

(c) $(3 - \log_3 3)^3$ **(d)** $\left(\frac{3 \log_x x + 2}{4 - 2 \log_5 1}\right)^2$

(e) $\log_2 (6 - 5 \log_7 7)$

6. Given that $\log_4 x = 2$ and $\log_2 y = 3$, evaluate $\frac{x}{y}$.

7. Given that $\log_3 x = a$ and $\log_{\sqrt{3}} y = b$, express xy^2 as a power of 3.

8. Solve the following simultaneous equations.

(a) $\log_x 16 = 4$ and $\log_2 y = x$ **(b)** $\log_y x = 2$ and $xy = 8$

***9.** Given that $\log_4 y = a$ and $\log_8 (2y) = b$, show that $2a = 3b - 1$.

10. If $\log_2 (\log_3 x) = \log_5 5$, find x.

[1.4]

1. Evaluate the following logarithms.

(a) $\log_2 4$ **(b)** $\lg\left(\frac{1}{10}\right)$ **(c)** $\log_2 8$

(d) $\log_3 27$ **(e)** $\log_2\left(\frac{1}{4}\right)$ **(f)** $\log_9 \sqrt{}$

2. Evaluate the following expressions.

(a) $\log_8 2 + \log_8 4$ **(b)** $\log_3 36 - \log_3 12$

(c) $\log_2 60 - \log_2 15$ **(d)** $\log_3 4 + \log_3 2 - \log_3 72$

(e) $\log_6 54 - 2\log_6 3$ **(f)** $\log_5 4 + 2\log_5 3 - 2\log_5 6$

3. Simplify and express each of the following as a single logarithm.

(a) $\log_a 8 - 2\log_a 4$ **(b)** $2\log_x 5 - 3\log_x 2 + \log_x 4$

(c) $\lg\left(\frac{8}{75}\right) - 2\lg\left(\frac{3}{5}\right) + 4\lg\left(\frac{3}{2}\right)$

(d) $2\lg(x+2) + \lg(x+1) - \lg(x^2+3x+2)$

4. Evaluate the following:

(a) $\log_a \sqrt{a}$ **(b)** $\log_a \frac{1}{a^3}$

(c) $\frac{\log_a 9}{2\log_a 27}$ **(d)** $\log_{\sqrt{a}} a^2$

5. Given that $\log_a 3 = 0.477$ and $\log_a 5 = 0.699$, evaluate the following:

(a) $\log_a 15$ **(b)** $\log_a 3\sqrt{5}$ **(c)** $\log_a 0.6$

(d) $\frac{\log_a 25}{\log_a 3a}$ **(e)** $\log_a(5a^2)$ **(f)** $\log_a\left(\frac{9}{5a}\right)$

6. Given that $\log_4 3 = a$ and $\log_4 5 = b$, express the following in terms of a and b.

(a) $\log_4 45$ **(b)** $\log_4 20$ **(c)** $\log_4 75$

(d) $\log_4(0.6)$ **(e)** $\log_4(0.75)$ **(f)** $\log_4(1.8)$

7. Given that $\lg x = p$ and $\lg y = q$, express the following in terms of p and q.

(a) $\lg(xy^2)$ **(b)** $\lg\left(\frac{10x}{y}\right)$ **(c)** $\lg\sqrt{10x^3y}$

(d) $\lg\left(\frac{100\sqrt{x}}{y^2}\right)$ **(e)** xy **(f)** $\lg(y^x)$

8. Find y in terms of x when

(a) $\lg y = 1 + 3\lg x$, **(b)** $\lg(y+1) = 2 - \frac{1}{2}\lg x$,

(c) $2\log_3 y - 4 = 3\log_3(x+2)$, **(d)** $3 + \log_2(x+y) = \log_2(x-2y)$.

9. If $\log_2(y+1) = 2\log_2 x + c$ and $y = 3$ when $x = 2$, find y in terms of x.

10. Express each of the following as a single logarithm.

(a) $2 + \log_3 5$ **(b)** $3 - 2\lg 5$ **(c)** $3\log_a 2 - 4 + \log_a a^3$

[1.5]

For questions 1 to 10, solve for x.

1. $\log_2 (x - 1) = \log_2 (4x - 7)$

2. $\log_3 (x + 2) + \log_3 (x - 2) = \log_3 (2x - 1)$

3. $\lg 18 + \lg\left(\frac{1}{3}x\right) - \lg (x + 1) = 0$

4. $\log_3 x + \log_3 (x + 2) = 1$

5. $2 \lg 5 - \lg (x + 2) = 1 - \lg (2x - 1)$

6. $\log_2 (x - 2) + \log_2 (8 - x) - \log_2 (x - 5) = 3$

7. $\log_2 (x - 1)^2 = 2 + \log_2 (x + 2)$

8. $\log_3 (x + 2) + \log_3 (10 - x) - 3 = 0$

9. $3 \log_x 2 + \log_x 18 = 2$

10. $\log_p 2 + \log_p (x - 1) = 0$

11. Evaluate.

(a) $\log_3 5 \cdot \log_5 27$ **(b)** $\dfrac{\log_5 4 \cdot \log_2 10}{\log_{25}\sqrt{10}}$

12. If a, b and c are positive numbers other than 1, show that

$$\log_b a \cdot \log_c b \cdot \log_a c = 1.$$

13. By using the substitution $y = \log_3 x$ or otherwise, solve the equation

(a) $\log_3 x + 2 = 3 \log_x 3$, **(b)** $\log_3 x^3 = (\log_3 x)^3$.

14. Solve the following equations.

(a) $\log_3 x = 9 \log_x 3$ **(b)** $4 \log_4 x - 9 \log_x 4 = 0$

(c) $\log_2 x = \log_4 (x + 6)$ **(d)** $\log_5 (5 - 4x) = \log_{\sqrt{5}} (2 - x)$

***15.** If $2 \log_a x = 1 + \log_a (7x - 10a)$, find x in terms of a.

***16.** Find x for which $27 \times 3^{\lg x} = 9^{1+\lg (x-20)}$.

[1.6]

In this exercise, give all answers correct to 3 significant figures where necessary.

1. Evaluate.

(a) $\lg 9$ **(b)** $\lg 24.3$ **(c)** $2 \ln 30$

(d) $\dfrac{4}{\lg 6}$ **(e)** $3 \ln 4 - \ln 6$ **(f)** $\ln 5 - \lg 3$

(g) $\lg 5 + 2 \lg 3$ **(h)** $\lg (\lg 90)$ **(i)** $(\ln 8)^2$

(j) $\ln (\lg 7)$ **(k)** $\dfrac{\lg 5}{\lg 2} + 1$ **(l)** $\dfrac{\ln 5}{\ln 2} + 1$

2. Evaluate the following by first converting them to common or natural logarithms.

(a) $\log_5 7$ **(b)** $\log_3 11$ **(c)** $\log_4 (5.3)$

(d) $\log_{\frac{1}{2}} 9$ **(e)** $\log_6 \pi$ **(f)** $\log_{2.5} (6.7)$

3. For each of the following, find y in terms of x.

(a) $10^y = x + 1$ **(b)** $e^{1-y} = 3x$ **(c)** $\ln (y + 1) = x$

(d) $2 \lg y = x - 2$ **(e)** $e^{2y} = x - 4$ **(f)** $\ln (x + y) - 4x = 0$

4. Solve for x.

(a) $\lg x = 0.61$ **(b)** $(\ln x)^2 = 3$ **(c)** $\ln x = \lg 2$

(d) $\lg 3x = 9$ **(e)** $\ln 2 \,.\, \ln 4x = 3$ **(f)** $\lg (x - 2) = (\lg 3)^2$

(g) $\ln 4x = \lg 3 \,.\, \lg 5$ **(h)** $\lg (2x + 1) = \log_2 3$ **(i)** $\lg (x - 1) = \ln (e^2 - 1)$

5. Evaluate $10^{\log_{10} 5}$, $e^{\log_e 4}$ and $3^{\log_3 7}$. Deduce the value of $4^{\log_4 3}$ and verify your answer using a calculator.

***6.** Solve the equations, giving answers correct to 4 significant figures.

(a) $\ln (\lg 3x) = 0.2$ **(b)** $\ln (4 - x) = \dfrac{1}{\lg 2} + \dfrac{2}{\lg 3}$ **(c)** $2 \log_3 x = \log_4 x + 1$

[1.7]

Solve the equations in questions 1 to 15.

1. $5^x = 9$ **2.** $(1.6)^x = 21$ **3.** $2(3^x) = 5$

4. $4 - 7^{2x} = 1$ **5.** $e^x = 7$ **6.** $e^{3x} = 14$

7. $4e^{2x} = 21$ **8.** $e^{4x} - 125 = 0$ **9.** $3^{x+1} = 12$

10. $4^{2x-3} = 20$ **11.** $e^{1+x} = 19$ **12.** $e^{\frac{1}{2}x} = 0.7$

13. $(4.1)^x = \pi$ **14.** $6^{\frac{2}{x}} = 4$ **15.** $e^{x^2} = 312$

16. Given $3^{x+1} \cdot 2^{x-2} = 21$, show that $6^x = 28$. Hence or otherwise, find x.

17. Find x, correct to 3 significant figures, in each of the following:

(a) $5^{x-1} \cdot 3^{x+2} = 10$ **(b)** $2^{2x} \cdot 5^{x+1} = 7$

(c) $4(3^{2x}) = e^x$ **(d)** $3^x \cdot 10^{2x} = 4 \cdot 20^{x-2}$

18. Using the substitution $y = e^x$, solve the following equations.

(a) $2e^{2x} - 3e^x = 2$ **(b)** $e^x = 7 - 12e^{-x}$ **(c)** $e^{3x} + 2e^x = 3e^{2x}$

19. Express the following in the form $\ln x = ax + b$ and find a and b.

(a) $x^3 = e^{6x-1}$ **(b)** $xe^{-x} = 2.46$ **(c)** $(xe^x)^2 = 30e^{-x}$

20. Given $y = 5e^{0.2x}$, find **(a)** y when $x = 3$, **(b)** x when $y = 12$.

21. Solve the simultaneous equations $\ln(3x - y) = 2\ln 6 - \ln 9$ and $\frac{(e^x)^2}{e^y} = e$.

22. Using suitable substitutions, solve for x.

(a) $9^x - 4 = 3^{x+1}$ **(b)** $2e^x = 7\sqrt{e^x} - 3$

Miscellaneous Exercise

1. Given that $y = 3(4)^{x+2}$, find, without using a calculator, the value of
(a) y when $x = -0.5$, **(b)** x when $y = 6$.

2. Given that $y = ax^b + 2$ and that $y = 5$ when $x = 2$ and $y = 29$ when $x = 4$, find the value of a and of b.

3. Given that $\log_3 2 = 0.631$, use the substitution $y = 3^x$ to solve the following equations.
(a) $3^x + 10 = 2(3^{x+1})$ **(b)** $9^x + 2(3^x) = 3^{x+2} - 12$

4. Given that $\log_2 x = a$ and $\log_8 y = b$, express x^2y and $\frac{x}{y}$ as powers of 2. Given further that $x^2y = 32$ and $\frac{x}{y} = 0.5$, find the value of a and of b.

5. Given that $\log_3 2 = 0.631$, evaluate $\log_3 6$, $\log_3 (2.25)$ and $\log_{\sqrt{3}} 6$.

6. Given that $\log_2 a = p$, express $\log_2 (4a^3)$ and $\log_8 \sqrt{a}$ in terms of p.

7. Given that $\log_2 x = p$ and $\log_4 y = q$, express the following in terms of p and q.
(a) $\log_2 xy$ **(b)** $\log_4 \frac{x}{y}$ **(c)** $\log_x 4y$ **(d)** x^2y

8. Given that $\log_b (xy^2) = m$ and $\log_b (x^3y) = n$, express $\log_b \frac{y}{x}$ and $\log_b \sqrt{xy}$ in terms of m and n.

9. Without using a calculator, solve the following equations.
(a) $(5^{x+1})^2 = 0.2\sqrt{5^x}$ **(b)** $\log_x 27 = 1.5$
(c) $\log_9 (3^{x+1}) = x^2$ **(d)** $\log_2 (\log_x 9) = 1$
(e) $\log_2 x \log_8 x = 12$ **(f)** $e^{4-x} = e^2 \cdot e^{x^2-4}$
(g) $\log_3 (x - 2) = 3 - \log_3 (x + 4)$ **(h)** $4^{3x} + \log_2 \left(\frac{1}{8}\right) = 5$

10. Solve the following equations.
(a) $3^{x+1} = 8$ **(b)** $e^{\frac{3}{x}} = 4$ **(c)** $\log_x 5 = 3$
(d) $\lg (\ln x) = 0.1$ **(e)** $5^x = e^{2x+1}$ **(f)** $\ln (e^{2x} - 5) = 2$

11. Find x such that
(a) $2e^x = 3 - e^{x+1}$, **(b)** $e^{-x}(2e^{-x} + 1) = 15$.

12. Given that $\ln 2 = a$ and $\ln 5 = b$, express $\ln \sqrt[3]{10e}$ in terms of a and b. Find also the number x such that $\ln x = \dfrac{b - 2a}{2}$.

***13.** Show that $3^{\lg x} = x^{\lg 3}$. Hence solve $2x^{\lg 3}(3^{\lg x}) - 5x^{\lg 3} = 3$.

14. The equation $2^{2x+p} - 2^{x+p} = 9(2^x) - 2$ has a solution $x = 1$. Find
(a) the value of p, **(b)** the other solution of the equation.

15. **(a)** Find x if $\log_2 x^2 - \log_2 (2x + 5) = 2$.
(b) $8^{10}\,5^{25}$ is a k-digit number, find k without using a calculator.

16. **(a)** The value, V dollars, of a car at age t months is given by $V = 40\,000e^{-kt}$, where k is a positive constant. The value of the car is expected to decrease to 30 000 dollars after 36 months. Calculate
(i) the value, to the nearest dollar, of the car when it is 15 months old.
(ii) the age of the car, to the nearest month, when its value is 20 000 dollars.
(b) Using the substitution $u = \ln x$, solve $\ln x^2 - 3 \log_x e = 1$.

17. **(a)** If $\log_2 k = 2 \log_2 6 + \log_2 10 - 3$, find k.
(b) Solve the simultaneous equations $8 \times 4^y = 2^{2x-1}$, $3^y\sqrt{3^x} = 81$.

18. **(a)** Solve the following equations.
(i) $\log_{x-2} (2x^2 - 10x + 13) = 1$ **(ii)** $2 \log_y 5 - \log_y 10 + \log_y 40 = 4$
(b) Given that $\lg (xy) - 2 = 3 \lg y - \lg x + \lg 4$, express y in terms of x.

19. Given that $2(3^{m+1}) = 5(6^n)$ and $4(9^m) = 7(6^{n-1})$, show that $3^m = \dfrac{7}{20}$ and $6^n = \dfrac{21}{50}$. Hence find m and n.

***20.** **(a)** Find the positive values of x for which $9x^{\frac{2}{3}} + 4x^{-\frac{2}{3}} = 37$.
(b) If $\lg 2 = m$, express $\log_8 5$ in terms of m.

***21.** **(a)** Solve the equation $\lg (3^x - 2^{4-x}) = 2 + \dfrac{1}{3}\lg 8 - \dfrac{1}{4}x \lg 16$.
(b) Without using a calculator, evaluate $(\lg 5)^2 + \lg 2 \lg 50$.

22. **(a)** Given that $p^n = 16p$, express $\log_2 p$ in terms of n.
(b) Without using a calculator, solve $4^x - 3^{x+\frac{1}{2}} = 3^{x-\frac{1}{2}} - 2^{2x-1}$.

***23.** **(a)** Given that $\ln y = 2 \ln (x - 1) + c$ and that $y = 20$ when $x = 3$, find the value of x when $y = 45$.
(b) Given that $a > b > 1$ and $2 \log_a b + 4 \log_b a = 9$, find b in terms of a.

***24.** **(a)** Solve for x in terms of a given that $\log_a 5 + 2 = \log_a (x + a) + \log_a (x - 3a)$.
(b) Find the exact value of x if $(3x)^{\lg 3} = (4x)^{\lg 4}$.

ANSWERS

Exercise 1.1 *(p. 2)*

1. **(a)** 1 **(b)** 4 **(c)** 144 **(d)** $\frac{1}{2}$ **(e)** 5 **(f)** $\frac{1}{3}$

2. **(a)** 54 **(b)** 4

3. **(a)** $8y$ **(b)** $\frac{1}{2}y^2$ **(c)** $\frac{12}{y}$ **(d)** $\frac{32}{y^3}$ **(e)** $\frac{1}{2}y^2$ **(f)** $y^3 - \frac{1}{y^2}$

4. **(a)** $18y^2$ **(b)** $\frac{1}{6}yz$ **(c)** $\frac{z^2}{y}$

Exercise 1.2 *(p. 5)*

1. **(a)** $\frac{3}{2}$ **(b)** $\frac{5}{2}$ **(c)** -2 **(d)** $-\frac{5}{2}$ **(e)** $\frac{1}{2}$ **(f)** 2
(g) 2, -2 **(h)** 2 **(i)** $\frac{2}{3}$ **(j)** $\frac{5}{4}$ **(k)** 2 **(l)** -6, 2
(m) -2, 4 **(n)** -2, 1

2. $a = 3$, $n = 2$

3. **(a)** $x = -\frac{4}{9}$, $y = \frac{1}{9}$ **(b)** $x = 2$, $y = 1$ **(c)** $x = 1$, $y = -1$

4. $m = 4$, $n = 3$ **5.** 3

6. **(a)** 0, 1 **(b)** 0, 2

7. **(a)** 0, 1 **(b)** 1, 3 **(c)** $-\frac{1}{2}$, $\frac{1}{2}$ **(d)** 0 **(e)** -2, 0 **(f)** 3

9. **(a)** 2 **(b)** ± 2 **(c)** 6 **(d)** 3 **(e)** 2 **(f)** 2

10. $r = 3$, $k = \frac{2}{3}$ **11.** $x = 1$, $y = -1$ or $x = 2$, $y = 1$ **12.** 4

Exercise 1.3 *(p. 9)*

1. **(a)** $4 = \log_2 16$ **(b)** $-2 = \log_3\left(\frac{1}{9}\right)$ **(c)** $2 = \log_{10} 100$
(d) $3 = \log_a y$ **(e)** $x = \log_2 p$ **(f)** $4 = \log_x(2 - k)$

2. **(a)** $5^3 = 125$ **(b)** $2^{-2} = \frac{1}{4}$ **(c)** $4^3 = 64$
(d) $x^4 = 3$ **(e)** $3^n = y$ **(f)** $2^{p+1} = 4y$

3. **(c)**, **(d)** and **(e)** are not defined

4. **(a)** 8 **(b)** 3 **(c)** $\frac{3}{2}$ **(d)** 5
(e) $-\frac{7}{16}$ **(f)** $\frac{3}{4}$ **(g)** 2, 4 **(h)** 4

5. **(a)** -2 **(b)** 2 **(c)** 8 **(d)** $\frac{25}{16}$ **(e)** 0

6. 2 **7.** 3^{a+b}

8. **(a)** $x = 2$, $y = 4$ **(b)** $x = 4$, $y = 2$ **10.** 9

Exercise 1.4 *(p. 14)*

1. (a) 2 (b) -1 (c) 3 (d) 3 (e) -2 (f) $\frac{1}{4}$

2. (a) 1 (b) 1 (c) 2 (d) -2 (e) 1 (f) 0

3. (a) $-\log_a 2$ (b) $\log_x\left(\frac{25}{2}\right)$ (c) $\lg\left(\frac{3}{2}\right)$ (d) $\lg(x+2)$

4. (a) $\frac{1}{2}$ (b) -3 (c) $\frac{1}{3}$ (d) 4

5. (a) 1.176 (b) 0.826 5 (c) -0.222 (d) 0.947 (e) 2.699 (f) -0.745

6. (a) $2a+b$ (b) $1+b$ (c) $a+2b$ (d) $a-b$ (e) $a-1$ (f) $2a-b$

7. (a) $p+2q$ (b) $1+p-q$ (c) $\frac{1}{2}(1+3p+q)$

(d) $2+\frac{1}{2}p-2q$ (e) 10^{p+q} (f) $q(10^p)$

8. (a) $y=10x^3$ (b) $y=\frac{100}{\sqrt{x}}-1$ (c) $y=9(x+2)^{\frac{3}{2}}$ (d) $y=-0.7x$

9. $y=x^2-1$

10. (a) $\log_3 45$ (b) $\lg 40$ (c) $\log_a\left(\frac{8}{a}\right)$

Exercise 1.5 *(p. 18)*

1. 2 **2.** 3 **3.** $\frac{1}{5}$ **4.** 1 **5.** $\frac{9}{8}$

6. 6 **7.** $-1, 7$ **8.** 1, 7 **9.** 12 **10.** $1\frac{1}{2}$

11. (a) 3 (b) 8 **13.** (a) $3, \frac{1}{27}$ (b) $1, 3^{\sqrt{3}}, 3^{-\sqrt{3}}$

14. (a) $27, \frac{1}{27}$ (b) $8, \frac{1}{8}$ (c) 3 (d) $-1, 1$

15. $2a, 5a$ **16.** 40

Exercise 1.6 *(p. 20)*

1. (a) 0.954 (b) 1.39 (c) 6.80 (d) 5.14 (e) 2.37 (f) 1.13
(g) 1.65 (h) 0.291 (i) 4.32 (j) -0.168 (k) 3.32 (l) 3.32

2. (a) 1.21 (b) 2.18 (c) 1.20 (d) -3.17 (e) 0.639 (f) 2.08

3. (a) $y=\lg(x+1)$ (b) $y=1-\ln 3x$ (c) $y=e^x-1$

(d) $y=10^{\frac{1}{2}x-1}$ (e) $y=\frac{1}{2}\ln(x-4)$ (f) $y=e^{4x}-x$

4. (a) 4.07 (b) 5.65, 0.177 (c) 1.35 (d) 3.33×10^8 (e) 18.9
(f) 3.69 (g) 0.349 (h) 18.7 (i) 72.5

5. 5; 4; 7; 3 **6.** (a) 5.550 (b) $-1\,829$ (c) 2.484

Exercise 1.7 *(p. 22)*

1. 1.37 **2.** 6.48 **3.** 0.834 **4.** 0.282 **5.** 1.95 **6.** 0.880
7. 0.829 **8.** 1.21 **9.** 1.26 **10.** 2.58 **11.** 1.94 **12.** -0.713
13. 0.811 **14.** 2.58 **15.** ±2.40 **16.** 1.86

17. (a) 0.633 (b) 0.112 (c) -1.16 (d) -1.70

18. (a) 0.693 (b) 1.10, 1.39 (c) 0, 0.693

19. (a) $a=2, b=-\frac{1}{3}$ (b) $a=1, b=0.9$ (c) $a=-1.5, b=1.7$

20. (a) 9.11 (b) 4.38 **21.** $x=3, y=5$

22. (a) 1.26 (b) $-1.39, 2.20$

Answers to Miscellaneous Exerci

1. **(a)** 24 **(b)** $-\frac{3}{2}$ **2.** $a = \frac{1}{3}$, $b = 3.17$

3. **(a)** 0.631 **(b)** 1, 1.262

4. 2^{2a+3b}, 2^{a-3b}; $a = \frac{4}{3}$, $b = \frac{7}{9}$ **5.** 1.631; 0.738; 3.262 **6.** $2 + 3p$; $\frac{1}{6}p$

7. **(a)** $p + 2q$ **(b)** $\frac{1}{2}p - q$ **(c)** $\frac{2}{p}(1 + q)$ **(d)** 2^{2p+2q}

8. $\frac{1}{5}(4m - 3n)$; $\frac{1}{10}(2m + n)$

9. **(a)** -2 **(b)** 9 **(c)** $-\frac{1}{2}$, 1 **(d)** 3

(e) 64, $\frac{1}{64}$ **(f)** -3, 2 **(g)** 5 **(h)** $\frac{1}{2}$

10. **(a)** 0.893 **(b)** 2.16 **(c)** 1.71 **(d)** 3.52
(e) -2.56 **(f)** 1.26

11. **(a)** -0.453 **(b)** -0.916 **12.** $\frac{1}{3}(a + b + 1)$; $\frac{\sqrt{5}}{2}$ **13.** 10

14. **(a)** 3 **(b)** -3 **15.** **(a)** -2, 10 **(b)** 27

16. **(a)** **(i)** 35 482 **(ii)** 87 **(b)** 0.368, 4.48
17. **(a)** 45 **(b)** $x = 4$, $y = 2$

18. **(a)** **(i)** $\frac{5}{2}$ **(ii)** $\sqrt{10}$ **(b)** $y = \frac{x}{20}$

19. $m = -0.956$, $n = -0.484$

20. **(a)** 8, $\frac{1}{27}$ **(b)** $\frac{1 - m}{3m}$ **21.** **(a)** 3 **(b)** 1

22. **(a)** $\frac{4}{n - 1}$ **(b)** $\frac{3}{2}$ **23.** **(a)** 4 **(b)** $\sqrt{a}$

24. **(a)** $4a$ **(b)** $\frac{1}{\ }$